Day 14 - Intro to Hypothesis Testing

Learning outcomes and terms to know

In this section, we explore how to formally make inference about unknown population parameters using a process known as hypothesis testing. By the end of this activity, you should know:

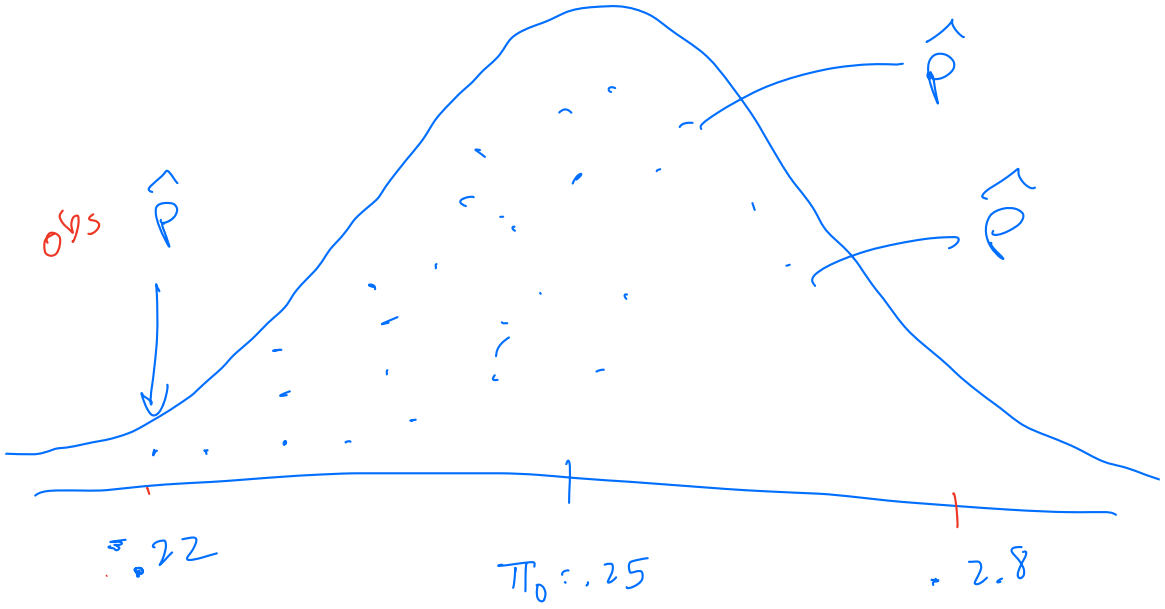
- null hypothesis
- alternative hypothesis
- null distribution
- p-value
- observed statistic
- parameter of interest

Warm-up

In May 2021, YouGov asked 4,839 adult Great Britain residents whether they think aliens exist, and if so, if they have or have not visited Earth. In the survey 1,065 responded "I think they exist, and have visited Earth", 1,355 responded "I think they exist, but have not visited Earth", 1,403 responded "I don't think they exist", and 1016 responded "Don't know".

Based on this survey, is there evidence to suggest that the **true proportion** of all British adults that think aliens exist and have visited earth is not equal to 0.25?

- π: true proportion of all GB adults that believe aliens exist & have visited Earth. (Parameter of interest)
- $\hat{p} = .22$; the sample proportion of GB adults that believe aliens exist & have visited Earth. (sample statistic)



Hypothesis testing

Note

The null hypothesis significance testing framework provides a mechanism to formally assess questions like the warm-up exercise. This framework is comprised of the following steps:

1. Frame the research question in terms of null (H_0) and alternative (H_a) hypotheses. Both hypotheses are always formed in terms of the parameters (population).
2. Collect data to address the hypotheses. Data are summarized by statistics (sample).
3. Model the randomness that would occur if the null hypothesis is true using either theory (CLT) or simulation methods. The way this is done depends on the question of interest and the **types of variables that are observed**. (This is when we find the sampling distr.)
4. Analyze the data by comparing the observed statistic to the distribution of statistics under the null hypothesis.
5. Form a conclusion by calculating the probability of observing your test statistic (or a statistic that is stranger than that) if the null hypothesis is true, called a p-value.

other parameters: $\pi_1 - \pi_2$, $\frac{\pi_1}{\pi_2}$, μ, $\mu_1 - \mu_2$, μ_3, β_0, β_1

Example: Aliens

1. Frame the research question in terms of hypotheses.

H_0: $\pi = 0.25$; the population proportion of EB adults who think aliens have visited Earth is 0.25.

H_a: $\pi \neq 0.25$; the population proportion of EB adults who think aliens have visited Earth is not equal to 0.25.

2. Collect data to address the hypotheses.

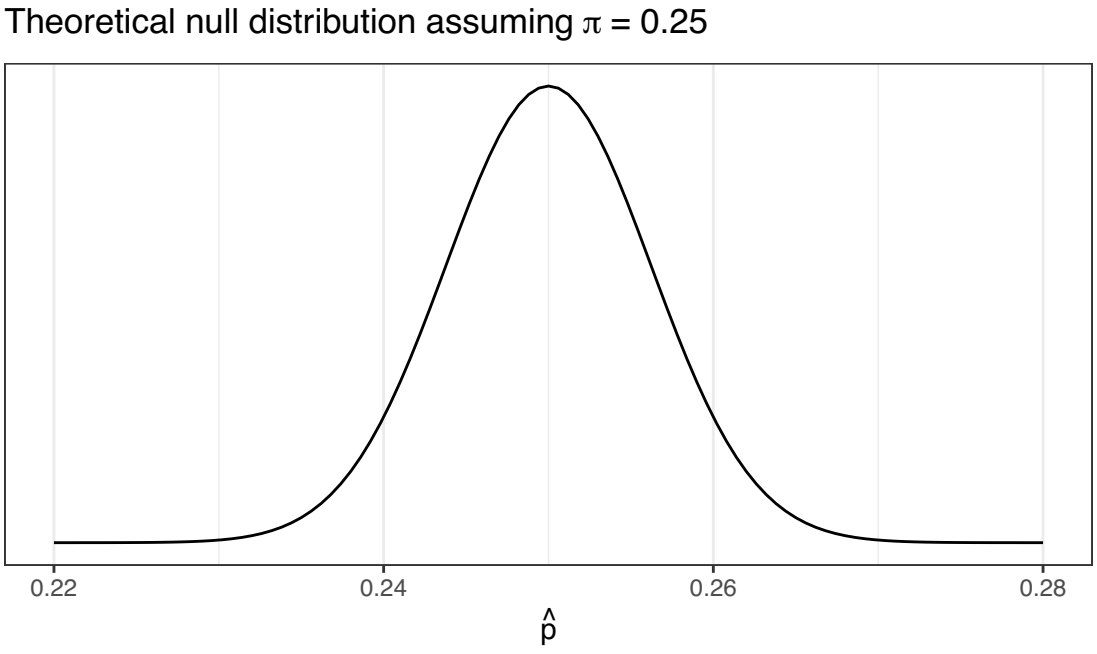
$\hat{p} = \frac{1065}{4839} \approx 0.22$

$\hat{p} = 0.25$

3. Model the randomness that would occur if the null hypothesis is true (using simulation).

3. Model the randomness that would occur if the null hypothesis is true (using the CLT).

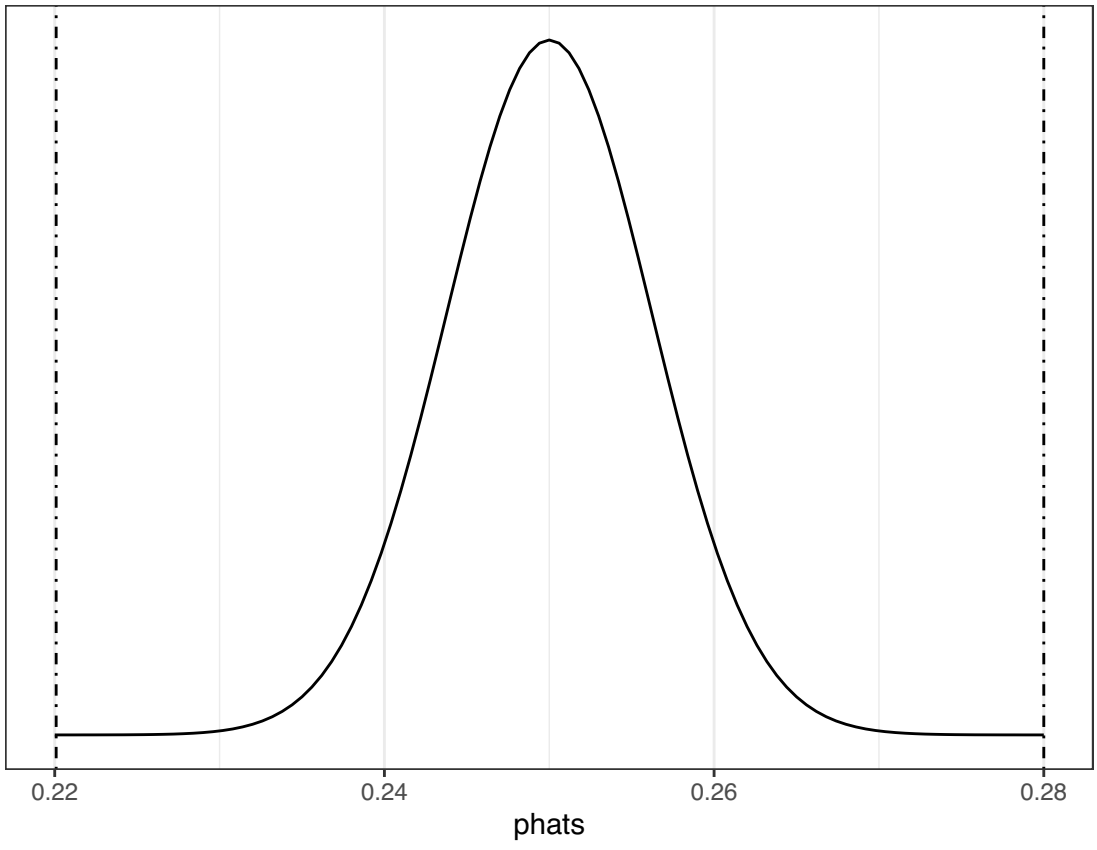
```
# according to the CLT
tibble(
  phats = seq(
    .22, .28,
    length.out = 100
  )
) %>%
ggplot(aes(phats)) +
  stat_function(
    fun = dnorm,
    n = 101, # not sample size, just a plotting parameter
    args = list(mean = .25, sd = sqrt(.25*.75/4839))
  ) +
  labs(y = "") +
  scale_y_continuous(breaks = NULL) +
  theme_bw() +
  labs(
    title = bquote("Theoretical null distribution assuming"~pi~"= 0.25"),
    x = bquote(hat(p))
  ) +
  xlim(c(.22, .28))
```



4. Analyze the data (using the simulated distribution).

4. Analyze the data (using the CLT).

```
# according to the CLT
tibble(
  phats = seq(
    .22,
    .25 + 4*sqrt(.25*.75/4839),
    length.out = 100
  )
) %>%
ggplot(aes(phats)) +
  stat_function(
    fun = dnorm,
    n = 101, # not sample size, just a plotting parameter
    args = list(mean = .25, sd = sqrt(.25*.75/4839))
  ) +
  labs(y = "") +
  scale_y_continuous(breaks = NULL) +
  theme_bw() +
  geom_vline(aes(xintercept = 1065/4839), linetype = "dotdash")+
  geom_vline(aes(xintercept = .28), linetype = "dotdash")
```



5. Form a conclusion.

p-value $< 1/5000$

If the true proportion of GB adults that believe aliens have visited Earth is actually 0.25, then is less than a $1/5000$ chance of observing a sample proportion of GB adults who believe aliens have visited Earth of 0.22, or something more extreme. Therefore, we would conclude that $\pi \neq 0.25$.